Best performances of a pop-pop engine

What follows is based on a simple math model. The first goal is to check if we can roughly predict the performances. Then, by comparing theoretical results with measured ones on some engines we will try to improve the math model in order to foresee the performances of future engines.

H1. First simplification: The pop-pop engine will be considered as a simple oscillator made of a spring of stiffness k (the gas) and a mass M (the water snake). The frequency of such an oscillator of the first order is defined by $F = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$

x being the displacement, F the force, P the pressure, S the cross section surface and V the gas volume one can write: $k = \frac{dF}{dx}$ $dF = dP.S$ $dP = \frac{P}{V}dV$ and $dV = Sdx$. The combination of these 4 equations allows to define the stiffness $k = \frac{PS^2}{V}$. And the mass is given by $M = \rho Sl$, l being the mean length of the water snake and ρ its specific gravity.

From this we get the general formulae: $\boxed{F = \frac{1}{2\pi}\sqrt{\frac{PS}{\rho Vl}}}$ which gives the frequency in Hz if legal units are used (P in Pa, S in m^2, ρ in kg/m^3 and l in m).

H2. Second simplification: The reciprocating movement of the water inside the pipe will be considered as sinusoidal. Even though it is not quite exact, we checked on various occasions that it is nearly correct and the influence of the *non-sinusoidality* (I take the responsibility of this wording) was in practice invisible on the thrust.

This simplification allows to determine the thrust of each pipe (See « Pop-pop engine and momentum theory »): $\boxed{T = \frac{\pi\rho}{16}(\pi dFh)^2}$ h being the stroke of the water snake.

H3. Third simplification: For the engines provided with a nozzle twill be considered that this latter has no length ; i.e. that the speed is faster, and the thrust as well, but that the nozzle doesn't change the amount of water in motion. We will note δ the diameter of the nozzle.

Additional notations:

V_0 being the volume of the boiler drum
L being the length of the pipe
S being its cross section surface
α being the average percentage of the pipe which is filled with gas. The length of the water snake will be L(1-α) and its stroke h will be at the best the lowest of the two values 2Lα and 2L(1-α).

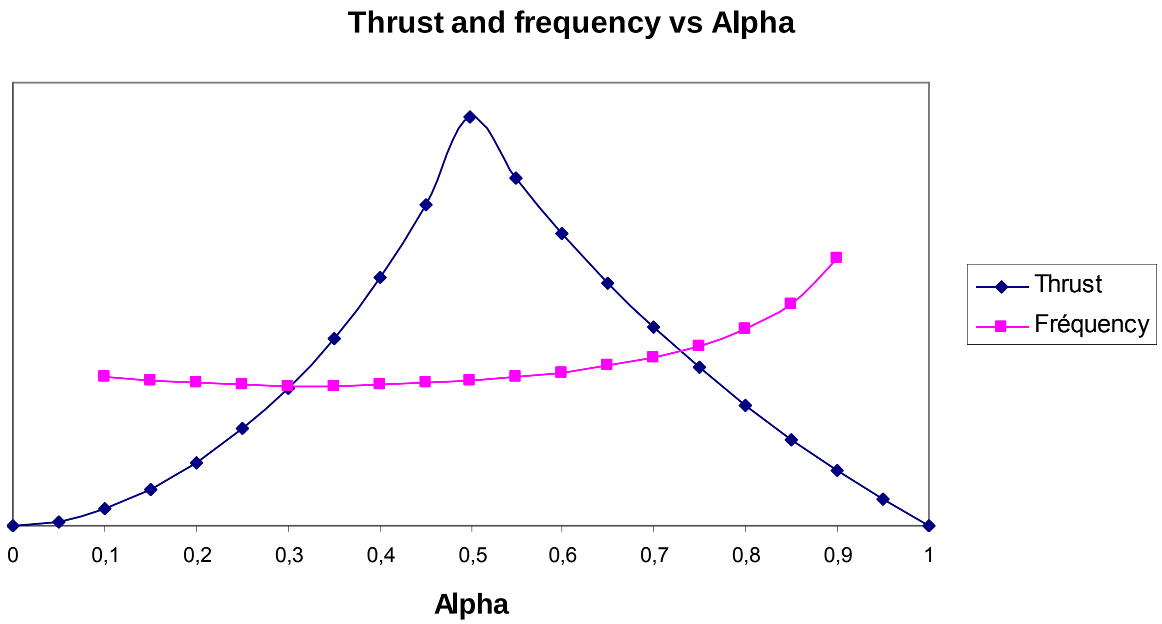
The following formula apply to any rigid pop-pop engine (i.e. not provided with a diaphragm).

$h = 2L\alpha$ if $\alpha < 0{,}5$ $h = 2L(1-\alpha)$ if $\alpha \geq 0{,}5$

Frequency: $F = \frac{5}{\pi}\sqrt{\frac{S}{(V_0 + SL\alpha)L(1-\alpha)\rho}}$

Thrust per nozzle: $T \leq \frac{\pi\rho}{16}(\pi dLF)^2(\frac{d}{\delta})^2$ We don't know the stroke ; that is why the inequality symbol exists in the equation. The equality would correspond to the maximum possible thrust.

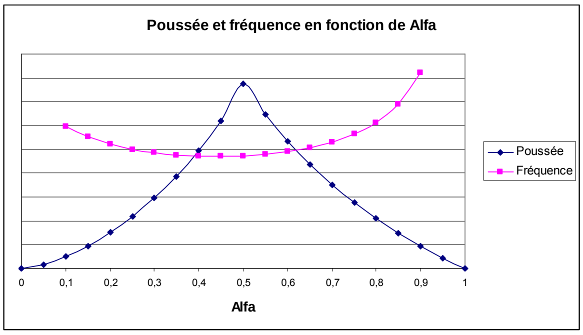
Using both previous formulas we can look at the influence of α on the frequency and on the maximum possible thrust. We applied them to several tenths of engines and all the curves that we got have the following shape:

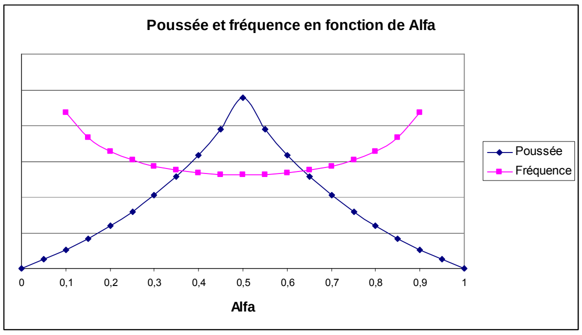


Note that T and F are represented versus α and not versus time, even though the engine generally starts at small values of α.

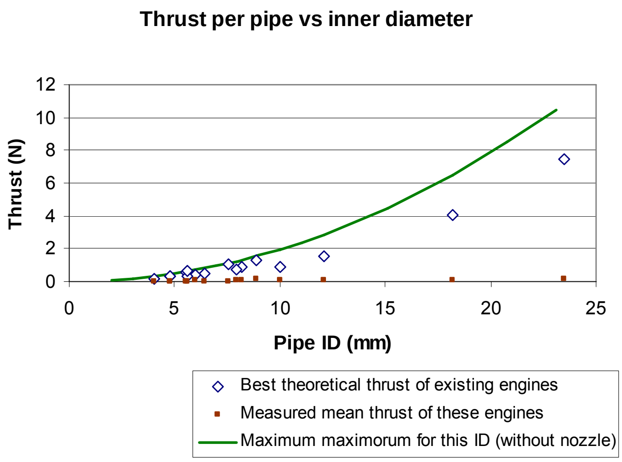
According to this model one can see (this is true for all the engines entered in the simulator) that the maximum thrust corresponds very closely to α=0.5; i.e. to a stroke of the water snake all along the pipe. On one hand, this is quite in accordance with what we observed on transparent engines. On the other hand, this justifies why the engines are more powerful once some air is introduced in the drum (see « Gas in a pop-pop engine ».

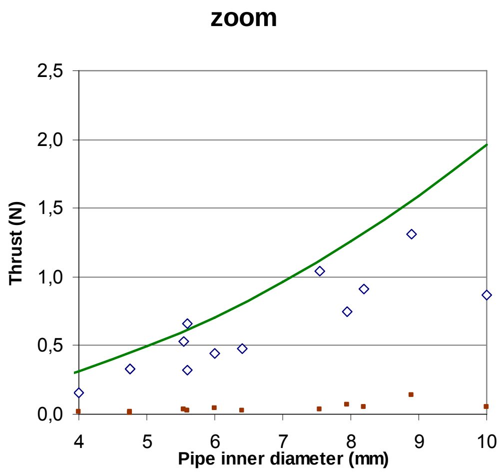
For what concerns the frequency, the curve versus α is always increasing when the drum volume is big. It is lower in the middle for the engines which have a small drum volume compared to the pipe one (graph on the left below). The most extreme case corresponds to a drumless engine. In this particular case, the graph has a symmetry axis for α =0.5 (graph on the right).





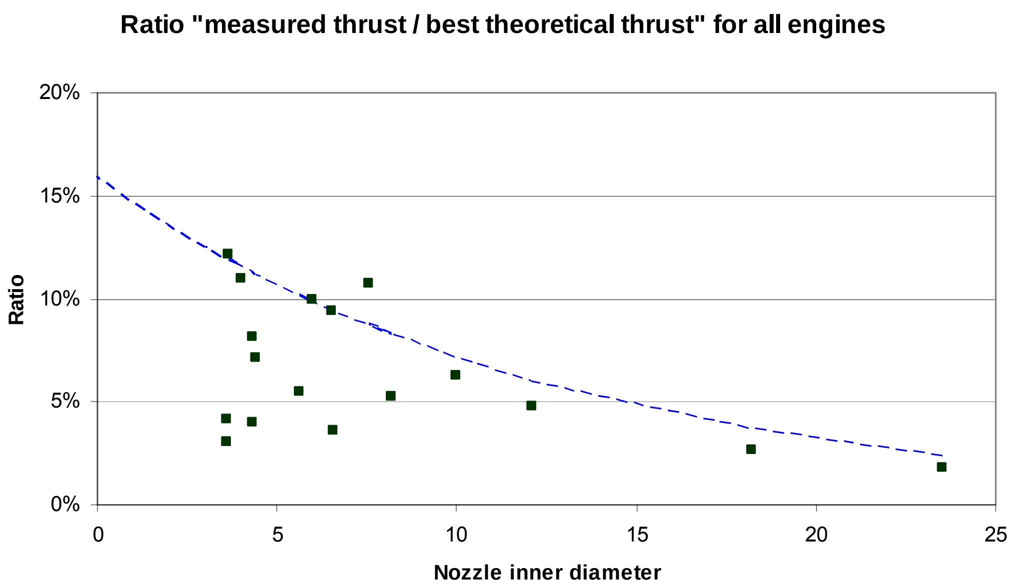
The math model that we used leads to defining the best engine as the one which has no drum. In practice we got that result on big engines, but it is difficult to run a small drumless engine during a significant time. As the theory bypass this kind of detail we have drawn the curve of the best thrust for different pipe diameters (green curve), and then we have plotted on the same graph the max thrust of some well known engines (diamonds) that we knew as performing. It is always below because all these engines are provided with drums. At last, we have plotted the mean thrust measured on each engine during a significant time (brown squares).





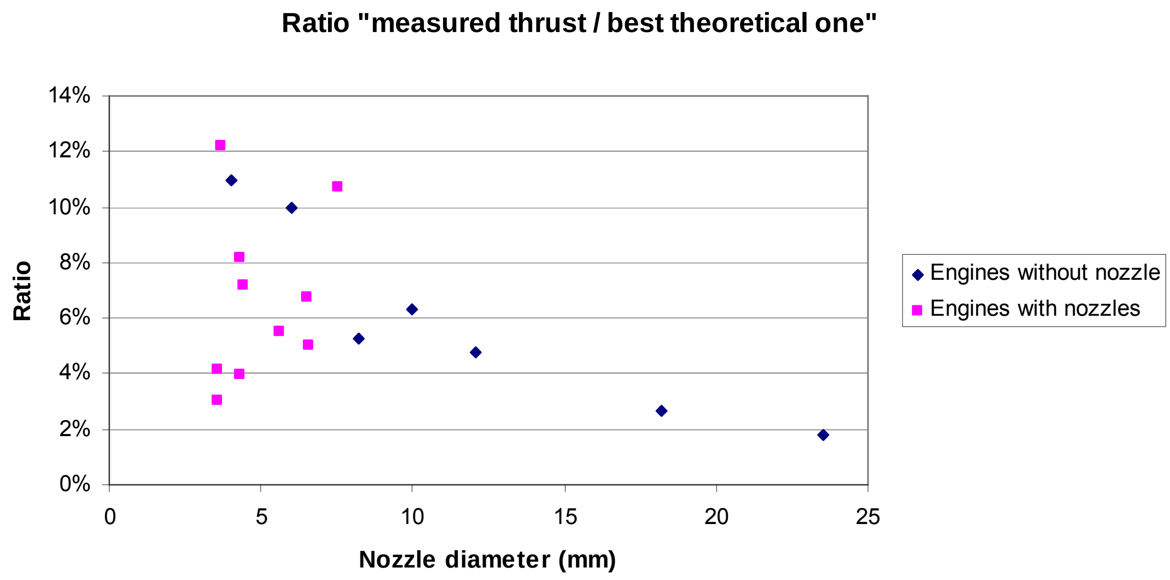
►All the performances are bad.
►Some engines are closer than others to the maximum possible.
►The bigger the diameter, the farther the results are from the maximum.

To see if a law could emerge we have calculated the ratio between measured thrust and maximum thrust that each engine could have exerted depending on its dimensions. If we place in abscissa the pipe diameter instead of the nozzle diameter there is a little difference. See below for instance the graph taking into account the outlet diameter.



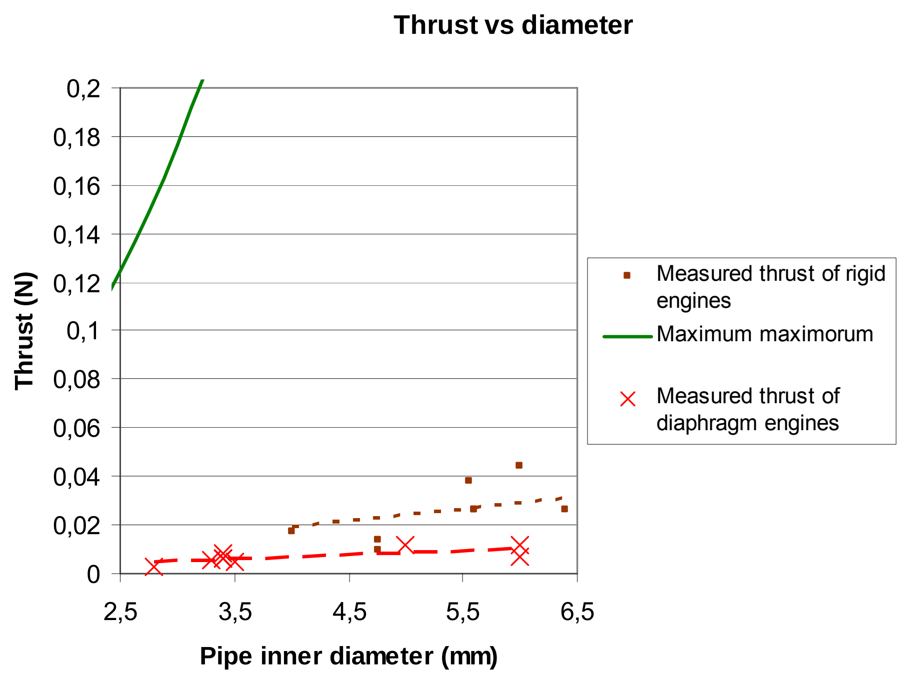
Uncertainties being taken into account, for the best engines there is roughly a law $\eta = 0.16e^{-0.08\delta}$.

Next graph shows the engines with nozzles (red squares) and the engines without nozzles (blue diamonds).



Looking at the curve and the dots we can assess two things:
►The engines provided with nozzles are not as good as we could have expected a priori.
►This new approach sets again as evident the fact that big engines are less interesting than small ones.

Though we are not able to model diaphragm engines, the maximum maximorum limit applies. Therefore, we have added on the graph the performances of some of them. The maximum maximorum is shown by the green curve, rigid engines by brown dots, diaphragm engines by red crosses.



Once again, this shows the superiority of rigid engines, however it still exists for them an enormous progress margin…