Frequency of a pop-pop engine

By Jean-Yves

According to some of my experiments the running of a pop-pop engine is easier, more stable, longer…when the pipe is long. But this is not the most interesting aspect. A long pipe can involve different frequencies and thrusts.

Experiments and computations show that the thrust increases with the product Frequency x Stroke volume. Hereafter we will look at how to control the frequency.

I believe that the pop-pop engine is an oscillator governed by Van der Pol equation. The heating power is only there to stimulate this oscillator. Unfortunately, we don't know the value of the parameters that would allow studying the Van der Pol model. Therefore, let's start from the simplest oscillator: a spring and a mass. In a pop-pop engine there is a spring which is the gas (steam and other gasses) located in the evaporator. And there is a mass that is the water snake which oscillates in the pipe. The gas volume and the water snake mass are always changing. Nevertheless, for each "cruising" conditions there is a mean value which should give us a rough estimate of the result.

Let's call V the mean gas volume, M the mean mass of the water snake, S the cross section area of the pipe and T the gas temperature (in °K). The natural frequency of the oscillator is approximately $F = \frac{1}{2\pi}\sqrt{\frac{nRTS^2}{MV^2}}$ (The formula will be justified further.)

When the gas volume increases (due to heating power increase or other changes) the water mass decreases because the water/gas interface climbs down in the pipe. When the interface is lower, on one hand the spring is softer and on the other hand the mass is lighter. Which one of these two factors takes precedence depends on the engine basic design. If the evaporator has a rather big volume, the frequency will be higher and the stroke will be longer. This latter is clearly visible for instance on Loïc's video of the glass engine. (www.eclecticspace.net). The one made of a test tube and a pipette.

As a first approach we can say that n evolves as V because PV=nRT and the engine works roughly at the atmospheric pressure and roughly at 373°K (100°C). Therefore we can simplify and write $F = \frac{k}{\sqrt{MV}}$ with $k = \frac{S}{2\pi}\sqrt{\frac{P}{R}}$ which is a constant for a given engine. Let's continue with Loïc's motor made of Pyrex glass. V doesn't change very much because the evaporator is big compared to the pipe volume, while M is roughly divided by 2 when the interface is in the middle of the pipe instead of being at the top. The ratio between the frequencies of these two extreme cases should be the square root of 2. Now look at my paper entitled *Gas in a pop-pop engine*. The result being roughly this one, it encourages going on our reasoning.

If you use big pipes and an evaporator with a small volume the result could be different. Perhaps a ratio of 2 or 3, but I cannot imagine that it could be very high because observation of transparent engines shows that the engine doesn't oscillate satisfactorily below a certain gas volume.

Assuming the hypothesis that the pop-pop engine is first of all governed by the resonance of the mass spring system, I made some calculations for engines without diaphragm.

To calculate the starting frequency, I assumed for simplification that all the water in the evaporator was converted into steam. That is roughly what we observe with transparent engines. The amplitude is small and the interface is located close to the evaporator.
For the frequency at full thrust I assumed that the interface was oscillating around a mean position located at the middle of the pipe.

The purpose of this study is not to get a result with several significant digits. It is only, to get a rough estimate.

Notations :

V=volume of the evaporator
d=diameter of the pipe
S=area of the liquid piston (cross section of the pipe)
L=length of the pipe
n=number of pipes
ρ=specific gravity of water (The mean temperature being low, ρ=1000kg/m^3 is used)
x=location of the interface inside the pipe
P=pressure in the evaporator

Avogadro law (P.V=n.R.T) leads us to write $dP = \frac{P}{V} dV$; hence, dV=Sdx and dF=SdP.

From that we get the stiffness $k = \frac{dF}{dx} = \frac{P.S^2}{V}$ and the frequency $\boxed{F = \frac{1}{2\pi}\sqrt{\frac{P.S^2}{M.V}}}$ with legal units (F in Hz, P in Pa, M in kg and V in m^3).

This allows the calculation of:

Starting frequency $F_1 = \frac{10d}{4}\sqrt{\frac{1}{\pi.V.L.n}}$

Frequency at full thrust $F_2 = \frac{10d}{4}\sqrt{\frac{n}{\pi \frac{L}{2}(V + \pi \frac{d^2 L.n}{8})}}$

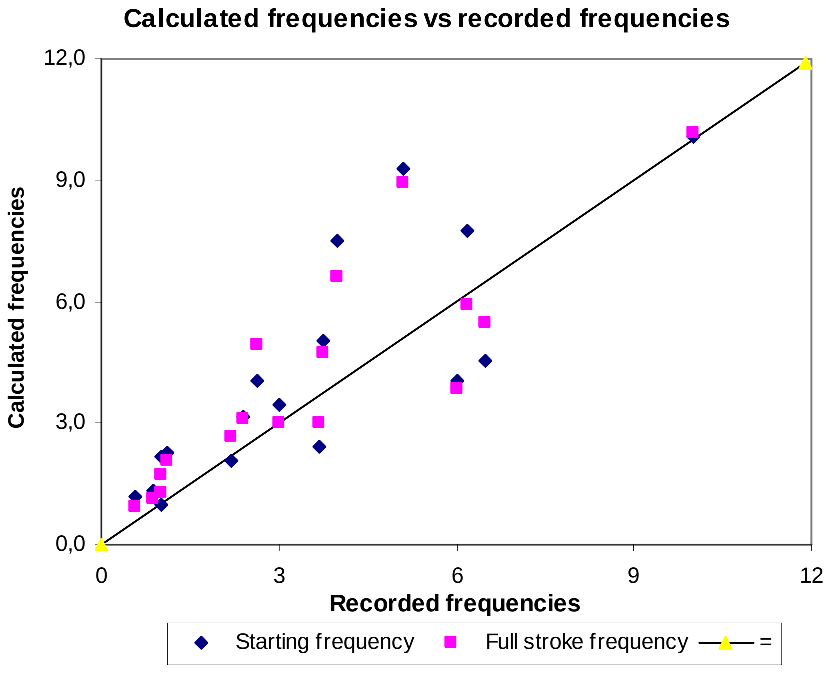
Let's calculate what could be the theoretical frequencies and let's compare with records of some of our engines (selected at random).

V (cc)	24,6	3,4	4,2	2	3,2	24,6	13,1	252	30,8	18,1	2,84
L (mm)	500	375	500	450	260	500	330	1060	820	840	220
n	1	1	1	1	1	1	1	1	1	1	2
d (mm)	5,2	6	5,2	5,2	6	6	6	11,1	12,1	9,5	4
F1 (Hz)	2,1	7,5	5,1	7,7	9,3	2,4	4,1	1,0	3,4	3,4	10,1
F2 (Hz)	2,7	6,6	4,8	5,9	9,0	3,0	4,9	1,2	3,0	3,0	10,2
F (Hz)	2,2	4	3,75	6,2	5,1	3,67	2,65	1	2,4	3,0	10

F1 and F2 are the theoretical frequencies. F is the recorded one (noted objectively, but not always accurately).

All these engines without exception (and they were not selected specifically to build this table) have a running frequency that is roughly in accordance with the calculated one. None of them reaches half or twice the value.

We can display the data of this table and the one of some other engines on a graph.



On this graph one can check that the frequencies are roughly as expected.

It can also be seen that the actual frequency is generally slightly lower than the calculated one.

We have not analyzed many engines, and the measurements are not accurate enough to get more from these data.

Conclusion:

- The knowledge of the dimensions of a pop-pop engine allows foreseeing an estimate of its running frequency.
- The frequency is approximately proportional to the pipe diameter and inversely proportional to the square root of the evaporator volume and to the one of the pipe length.
- In practice on performing engines the evaporator volume is small and the ratio length/diameter doesn't evolve much with the size of the engine. Therefore, when running is optimal **the frequency is approximately inversely proportional to the diameter** of the pipe(s).

$$F \approx \frac{25}{d}$$ with F in Hz and d in mm.

Note: this document doesn't apply to diaphragm engines. Such engines operate at lower or very lower frequencies than same engines provided with rigid wall instead of diaphragm.