Pop-pop engine and Helmholtz resonator

I want to kill an erroneous rumor! The pop-pop engine is not – as we generally use it – a Helmholtz resonator.

I have got (recently and accidentally) the Helmholtz resonator when repairing a brazed joint. Therefore, it could exist. This confirmation was as happy as unexpected (*1). But this resonator – as I described it in 2005 (*2) from a theoretical point of view – has a fundamental frequency which is much higher than the one of the pop-pop. As I'm not a music specialist I would say that the frequency was approximately 1 kHz. What I can guarantee is that it was clearly audible, far above 150Hz and far below 10 kHz. As the dimensions of the singing engine are well known, we can do some calculations.

Volume of the drum: V=1838mm^3
Cross section of the pipe: S=20mm^2
Length of the pipe: L=210mm
The temperature of the drum and the one of the air inside it during this musical concert was decreasing from approx 500 to 200°C (approx 950 to 400°F); which gives for the numerical application $f = \frac{c}{2\pi}\sqrt{\frac{S}{VL}}$ (lengths in m) ; i.e. 800>f>500Hz. This is quite in accordance with what was heard.

This same engine has a pop-pop frequency of approx 6Hz (measured between 5.8 and 6.25). Therefore, there is a ratio of about 100 between both frequencies which are completely disconnected one from the other one. We have to search elsewhere the reason of the pop-pop.

According to the metal surface temperature, we know that in normal operating conditions the steam-water interface is just at the outlet of the drum. On one side there is a spring made of steam at a mean pressure that is very close to the atmospheric pressure. And on the other side there is a mass which is the water contains inside the pipe. We can do some other calculations.

For our application, the mass of the water contains in the pipe was approx 4.2g and the volume of steam inside the drum 1.838 cm^3. At 100°C (212°F) the specific gravity of the steam is 0.60 kg/m^3. Consequently, there is $1.838x10^{-6}x0.60 = 1.102x10^{-6}kg$; i.e. $\frac{1.102x10^{-6}}{18x10^{-3}} = 61.2x10^{-6}$ mole. Assuming there is neither condensation nor vaporization, the steam plays the role of a spring of stiffness k such as $k = \frac{dF}{dl} = \frac{S.dP}{dl}$ with dP=variation of pressure related to the displacement of the water "sausage" in the pipe. Steam being considered as a perfect gas, PV=nRT.
For this application PV=61.2x10^{-6}x8.314x373 =0.19.
$k = \frac{dF}{dl} = \frac{SdP}{dl}$ ..and V=S.l (l=length of a imaginary drum of cross section identical to the one S of the pipe); which gives $P = \frac{nRT}{S.l}$ the derivate of which versus l being

$$P' = \frac{dP}{dl} = -\frac{nRT}{S.l^2} = -\frac{nRT}{S}x\frac{S^2}{V^2}$$

Note: The sign – comes from the fact the pressure increases when the volume decreases, but in fact it is the absolute value which concerns us.

$$k = S.P' = -\frac{nRTS^2}{V^2} = -\frac{61.2x10^{-6}x8.314x373x(20x10^{-6})^2}{(1.838x10^{-6})^2} = 22.47N/m$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{22.47}{4.2x10^{-3}}} = 11.6Hz$$

This is nearly the double of the real pop-pop frequency. But remember that the purpose was just to get a rough idea of the magnitude by using simplifying hypotheses.

*1. Circumstances of this accidental experiment.

The pop-pop engine was set vertically with the drum upside. The pipe was slightly squeezed in the lead jaws of a vise in order to repair a leaking brazed joint on the drum. The temperature was high to make the brazing metal melt. (Copper and silver alloy. Approx 630°C=1166°F). The welding torch was lit and doing some noise. About 10 seconds after the torch was removed from the drum, the brazed joint having a good aspect, the torch was switched off. Immediately I heard the sound emitted by the engine. I must confess I was not prepared to hear that and it took me some seconds to understand where it was coming from. Then I listened to it carefully until it vanished.

*2. See “Propulsion of a boat by means of a pop-pop engine” (called "First approach" on www.eclecticspace.net) , chapter 13, §1. March 2005.