Pop-pop propulsion and momentum theory

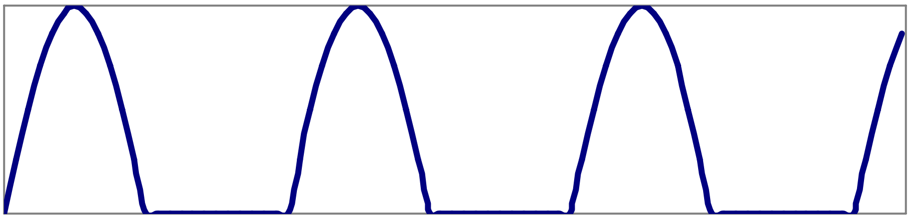
By Jean-Yves

A) Working at bollard pull conditions

The thrust of a waterjet is given by the formula $T = Q.V$, Q being the mass flow, and V the velocity of the water going out of the nozzle. We have checked experimentally by means of pop-pop pipes of various diameters that the momentum theory applies to direct waterjet propulsion. Obviously, it applies too to a reciprocating waterjet, but the calculation of T is more complicated because Q and V depend on the time. However, the momentum at any time can be defined mathematically.

Additional tests allowed us to check that the pop-pop propulsion looks (regarding flow) like the positive part of a sine function (see “Pop-pop engine and electrical analogy” and "The water snake movements").

Therefore, the flow Q versus time can be represented as follows.



The simplest way is to examine the function over a period 2π, to integrate it and to divide the result by 2π.

$T_{moy} = \frac{1}{2\pi}\int_0^{2\pi} Q_{(\theta)}V_{(\theta)}d\theta$ On one hand, this function can be split in two intervals (0-π and π-2π), the function being equal to zero in the second one. On the other hand, both Q and V vary, but they are linked together by the nozzle diameter.

Let’s assume q is the volume flow, s the nozzle cross section area, and d its diameter. $V_{(\theta)} = \frac{q_{(\theta)}}{s} = \frac{4.q_{(\theta)}}{\pi d^2}$ And the mass flow Q is linked to the volume flow by the specific gravity. Q=ρq.

In practice, the water is cold and its specific gravity ρ is 1000 kg/m^3.

Knowing that $q_{(\theta)} = \pi.C.F.\sin\theta$ with C=Stroke volume and F=Frequency, one can write:

$$T_{moy} = \frac{1}{2\pi}\int_0^{\pi}(\pi CF\rho\sin\theta)(\frac{4.\pi CF\sin\theta}{\pi d^2})d\theta = \frac{2C^2F^2\rho}{d^2}\int_0^{\pi}\sin^2\theta.d\theta$$

The peak value is $T_p = \frac{\pi^2\rho}{S}C^2F^2$ or $T_p = 4\pi\rho(\frac{CF}{d})^2$ when the flow has a circular section.

And the mean one is $T_{moy} = \frac{\pi^2\rho}{4.S}C^2F^2$ or $T_{moy} = \pi\rho(\frac{CF}{d})^2$

This formula shows that the <u>mean</u> thrust is proportional to the square of the product CxF which corresponds to a flow, and it is inversely proportional to the nozzle area. These trends are in accordance with the results of our many experiments.

The same thrust can be got with a direct flow Q such as $T = \frac{4}{\pi}\rho(\frac{Q}{d})^2$. This allows to know the relation between Q and CF: $\boxed{Q = \frac{\pi}{2}CF}$ …which confirms (very well!) the practical results of the test bench (see below).

1°) Coefficient π/2=1,57

The measurements were done in early 2006 without any a priori on the result we would get, except it had to be something between 1 and 3. We got Q=1.55CF rounded to 1.5 because of the measuring uncertainties. (See "Thrust measuring test bench"). 1.55 for 1.57. Experiment and theory confirm each other.

2°) Numerical value

The first test bench was only a comparative test bench. Later, when we measured the thrusts they were bigger than the theoretical value because of the concavity of the target. Since that time, we have done –with direct flow as well as with alternative one- measurements by means of a flat target. All the thrust measurements were seen higher than the expected value by approximately 10% to 40% depending on the test means (size of the target, size of the test tank, size of the nozzle, accuracy of the measuring tools…). It is very likely due to a recirculation of the water into the tank. Indeed, with direct flow the water entered in the tank on the front side of the target and the overflow was more or less on the opposite side. And, with alternative flow, we had the opportunity to observe the movement of some impurities which were running along a pseudo-ellipse on both sides of the nozzle, going towards the target, and coming back by the outside. See "Why do we measure more than the theory?" Since that time, even with a permanent flow we have observed the same phenomenon and we are convinced that there is the cause of the excessive measured value. It is good to know that, but the principle remains applicable; and particularly for comparative measurements.

3°) Effective value

By analogy with effective values in electricity, we tried to go further in this study, and we searched for a permanent flow that would involve the same power as a reciprocating one.

At the nozzle outlet, there is only kinetic energy: $P = \frac{1}{2} Q_m V^2$; which gives :

With direct flow: $P_c = \frac{\rho}{2s^2} Q^3 = \frac{8}{\pi^2} \rho \frac{Q^3}{d^4}$

With alternative flow: $P_{eff} = 4\rho \frac{(CF)^3}{d^4} \int_0^{\frac{1}{2F}} \sin^3 \omega t = \frac{\rho}{s^2} . \frac{\pi^2}{3} (CF)^3 = \frac{16}{3} \rho \frac{(CF)^3}{d^4}$

Comparing both equations, we get $Q = \sqrt[3]{\frac{2\pi^2}{3}} C.F$, i.e. Q=1,87C.F…but we know that $Q = \frac{\pi}{2} CF$; hence, the kinetic energy (or more precisely the kinetic power) is not the same in both cases.

To get the same thrust, with a reciprocating flow we need to supply a kinetic power higher by 70% than the one which would be needed with a direct flow. By itself, this small demonstration condemns the industrial use of a pop-pop engine. But it is not enough to stop us in this study.

4°) Mean velocity.

As the movement is periodic, we will call v_0 its mean velocity. $v_0 = \frac{.C.F}{.S}$.

Note: This is valid whatever the shape of the signal, and obviously it works with our sine wave signal.

B) Propulsion of a moving boat

We know how does a waterjet work at bollard pull conditions, but what about it when the boat is going forward? By analogy with what we did for bollard pull, we are going to split the propulsive and relaxing phases.

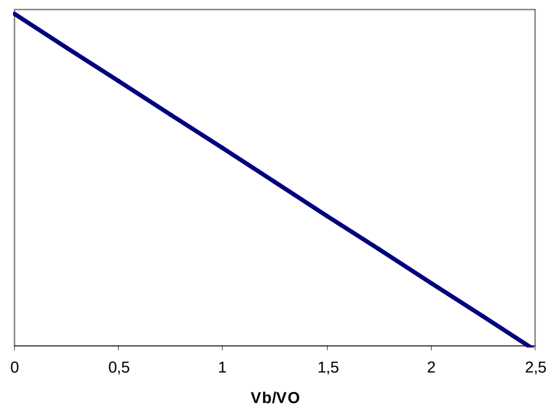
Notations:

V_j is the instantaneous velocity of the jet at the nozzle outlet (seen from the boat)
V_b is the boat speed (seen from a fixed reference)
S is the cross section area of the nozzle.

During the relaxing (or sucking) phase, water is taken from outside. Seen from the boat, this water has an initial velocity V_b and once in the boat it becomes steady (always seen from the boat). The sucked mass flow is $Q_m=\rho S V_j$, and the corresponding instantaneous braking force is $f_f = -\rho.S.V_j.V_b$.

During the propulsive phase, the waterjet delivers exactly the same thrust as at bollard pull conditions. Indeed, seen from the boat nothing differs: steady still water is taken and sent away at velocity V_j. The corresponding instantaneous propulsive force is $f_p = \rho.S.V_j^2$.

As the water movement is sinusoidal, the mean thrust is got by integrating f_f+f_p on a full period ; which gives : $\boxed{T_{moy} = \rho.S.v_0.\left[\frac{\pi^2}{4}.v_0 - V_b\right]}$



Hence, for a given pop-pop engine (v_0 or C, F and d known) we see that the thrust decreases linearly when the boat velocity increases. The thrust versus V_b/v_0 can be represented on a graph.
And we can check (because it was guessed) that the thrust finishes to becoming null. The corresponding boat velocity is $V_b = \frac{\pi^2}{4} v_0$

The power received by the boat is the product of the thrust by its velocity. $P_b=T_{moy}.V_b$. Its representation versus the boat velocity is given on the right. It is a parabola. Its derivate is zero for $V_b/V_0=\pi^2/8$.
Therefore, a sinusoidal pulsed waterjet transmits the maximum power when it propels the boat at approximately 1.2 times (theoretically $\pi^2/8$) the mean jet speed. And the corresponding power is $P_{max} = \rho.S.\frac{\pi^4}{64} V_0^3$