To justify this title : In several documents I used the words "water sausage" to describe the quantity of liquid water moving inside a pop-pop tube. In some documents written by Anglo-Saxons I read "water snake". Due to the idea of movement I feel it is better and from now I will use the words water snake.

Measurements of Pmax and Pmin in the boiler of a pop-pop engine showed that the cycle was not symmetrical. The absolute value (in math sense) of the max effective pressure (in the physical sense) is bigger than the one of the min effective pressure. Therefore, the electrical analogy using a sine function (See "Pop-pop engine and electrical analogy") is to be updated.

Several observations with impurities or with an object of density 1 voluntarily inserted inside a transparent tube showed that the visible interface was moving as the whole water snake. This is understandable because vaporization or condensation occurring at the interface concerns only a tiny amount of water. Hence, as a rough estimate we can consider that the water snake is like a piston on which the steam pressure acts; this pressure being alternately positive and negative.

## 1. General math approach :

Notations: $\quad$ f for steam pressure
S for tube cross section
M for the water snake mass
$\chi$ for acceleration
V for the snake velocity
y for its location

$$
\mathrm{V}_{(\mathrm{t}+\mathrm{dt})}=\mathrm{V}_{(\mathrm{t})}+\chi^{2} \cdot \mathrm{dt} \quad \mathrm{y}_{(\mathrm{t}+\mathrm{dt})}=\mathrm{y}_{(\mathrm{t})}+\mathrm{V}_{(\mathrm{t})} \cdot \mathrm{dt}
$$



To get a primitive of a periodical function which is itself periodical the positive ( + ) and negative (-) areas must be equal.

## 2. Sine model :

The sine pressure model (that we will name number 1) was studied in a report that you could read. This model being too simplified we are going to examine some others.

In order to get an evident dissymmetry on the models we will choose a ratio of 3 between |Pmax| and |Pmin|.

## 3. Model number 2: square signal :

The ratio 3 that we chose on the ordinate axis involves a cyclic ratio of $1 / 3$. Everything can be easily displayed on graphs. First graph = pressure. Second one = velocity.


And finally the water snake movement.


It can be seen that in spite of very hard hypotheses (square signals) the displacement is not very far from a sine function.

Note for purists:
To get a final representation in accordance with the physical reality, i.e. to display the top dead centre (TDC) at the top of the curve, we have inverted the sign of the last function.

The cycle is not symmetrical according to the x axis, but there are y axis symmetries ( $y=\Pi, 2 \Pi, 3 \Pi, \ldots$ ). It means that for every position of the water snake the outgoing speed is equal to the sucking one.
4. Model number 3: triangle (saw teeth) :


Note: We are not looking for the fifth decimal. This is only a rough and quick estimate. On this design the leading edge is not quite vertical and the lower triangle is not perfect. On one hand this is due to a coarse calculation. The period (2П) has been divided in only 24 slices for the numerical integration. On the other hand, why not a trapezoid instead of a triangle?...

The graphical result for the water snake position is there too not very far from a sine function.


As for the previous model, the sign has been inverted to display the TDC at the top of the curve.

In caricaturing one can say that in both cases the water spend more time at the bottom than at the top; which is not surprising because the low pressure which brings it up is weak.

## 5. Other approach :

Other approach: how is it possible to generate mechanically (by modifying the existing test bench) an asymmetric movement? We are going to examine three alternatives, and for each one the chronological way will be the opposite of the previous ones: $1^{\circ}$ ) position, $2^{\circ}$ ) velocity, $3^{\circ}$ ) acceleration or pressure.

### 5.1. Model number 4 :

Transverse shifting of the rotation axis compared to the piston one.



This graph of the pressure shows an interesting dissymmetry.

### 5.2. Model number 5 :

Connecting rod with a sliding shoe.



Not realistic in regard to the shape of the pressure signal.
5.3. Model number 6 :

Short connecting rod.



This model number 6 seems close to the reality (as we suppose it). The movement (position) is close to a sinusoid and as a first approximation the acceleration is plausible. However it is unlikely that the real movement is symmetrical according to the $y$ axis.

## 6. Modification of the bench :

The long connecting rod is needed for the pump itself. Adding a sliding shoe would involve too much play with the tools available. An additional articulation will be preferable.


Towards
piston
We have studied the function $\beta=\mathrm{f}(\alpha)$ for different values of $\mathrm{r}, \mathrm{d}, \mathrm{L}$ and X and we have selected the following ratios: $\mathrm{L} / \mathrm{r}=2,5$; $\mathrm{X} / \mathrm{r}=3$; $\mathrm{d} / \mathrm{r}=2$.

Béta (degrés) en fonction de Alpha


One can see that the absolute value of the extreme low pressure is half the one of the high pressure. And with a bit of imagination (by rotating the motor axis in the other way; i.e. by reading the graph from right to left) one can see that the pressure rise is fast. Both criteria correspond to what we are looking for.


## Application:

The full angular movement of the oscillating arm will be $47.2^{\circ}$. To get a compromise between inertia (to allow fast rotation) and relative plays, we have selected a radius $\mathrm{r}=16 \mathrm{~mm}$; which defines $\mathrm{X}=48, \mathrm{~L}=40$ and $\mathrm{d}=32$. And to get the piston stroke we are interested by, we have used the oscillating arm as a rocker arm with a lever R such as $8=$ R. $\operatorname{Sin}(47.2 / 2)$; i.e. $\mathrm{R}=20 \mathrm{~mm}$. In the extreme positions, the horizontal projection of the small arm is $20 \cdot \cos (47.2 / 2)=18 \mathrm{~mm}$. Between 18 and 20 we have selected 19 in order to make the system as linear as possible. (We have studied the function $\sin (\beta)$ between $-23.6^{\circ}$ and $+23.6^{\circ}$. The non linearity doesn't exceed 1.35\%)

Shortening of the long connecting rod by $\sqrt{48^{2}-19^{2}}=44 \mathrm{~mm}$
Fitting of an axis at 44 mm below C and at 19 mm on the left. (On the left to take into account the rotating direction).

Articulation on this axis of a rocker arm with one arm 40 mm long and the other arm of 20 mm doing with the first one an angle of $21.6^{\circ}$ (For this report I forget the details of trigonometry).


All what precedes relates to the period 2005-2006, and end of 2006 a simulator was designed and built based on these assumptions. (See "Hydraulic simulator of pop-pop engine".) End of 2009 it occured that I had worked for nothing because the actual movement is very very close to a sine function as proven by the following diagram recorded on Dec.5, 2009.

## Volume de gaz en fonction du temps (Gas volume vs time)



- V mesuré (recorded) — Sinusoïde parfaite (Perfect sine function)

The blue dots were recorded with a data logger (see "Experimental PV diagram of pop-pop cycle"). The red curve is a perfect sine function which has been superimposed. For this graph we used a big scale in order to set as evident the fact that the gaps between dots and curve are insignificant.

This result validates some previous studies such as « Pop-pop propulsion and momentum theory » which were based on the assumption of a sine movement.

